Spatial Monopoly Theory in 1885: Wilhelm Launhardt*

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Abstract: This paper attempts to demonstrate that the first appearance of the spatial monopoly model is in Launhardt's 1885 book. It shows that the theory of spatial demand and spatial monopolistic firm's economic decision was highly advanced in Chapter 27 of Launhardt's book entitled *Market Area for the Sale of Goods*. By his mathematical presentation of the spatial monopoly model, Launhardt anticipated the spatial monopoly analysis developed and extended by contemporary spatial economists. Launhardt's name deserved to be mentioned as an early precursor of modern spatial economics.

JEL classifications: D42, R32, B13.

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1. INTRODUCTION

The use of spatial monopoly model to analyze the impact of space and transportation cost on the consumer and producer economic decisions has produced many faceted trails of important developments in spatial pricing policy, market area and industrial location. The purpose of this paper is to show that the seeds of these developments were presented in Wilhelm Launhardt's masterpiece, *Mathematical Principles of Economics* (1885, 1993). Unlike other nineteenth century writers concerned with the economic decision of profit-maximizing monopoly in the non-spatial setting, Launhardt explicitly derived the algebraic form of spatial demand function and set up a formal spatial monopoly model to investigate the determination of mill price. Unfortunately, Launhardt's insights into the problems of spatial monopoly have not been explored to any degree, owing probably to inaccessibility of his writings[1] and by the fact that his analysis is severely restricted by the use of specific function forms, in particular the use of a quadratic utility function.[2] But the analysis of Launhardt's contribution to the theory of spatial monopoly appears warranted, especially as Schumpeter claimed that "[Launhardt's] almost ruthless use of particular forms of function – by which he produces results of disconcerting definiteness

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[1]. Only two Launhardt's books, *The Principles of Railway Location* (1900, 1902) and *Mathematical Principles of Economics* (1985, 1993), and a short excerpt of Chapter 32 of 1885 book (1968) exist in published translations. It may be noted that *The Principles of Railway Location* was out of print long time ago.

[2]. Niehans suggested that "Launhardt's addiction to special functional functions, particularly quadratic utility functions, often results in spurious precision, limited generality and reduced lucidity, but the basic contributions are sound, important and original." (1987, p.141). In their comment on Launhardt's chapter 32 in 1885 book, Baumol and Goldfeld pointed out that "linear demand and cost functions are consistently employed without any comment on their lack of generality. In this case the author pays a substantial penalty for his self-indulgence. For his linearity assumption seems to have just barely prevented him from being the first advocate of marginal cost pricing." (1968, p. 27).

– should be studied and improved rather than condemned *a limine.*" (1954, p. 948, n.10), and Blaug pointed out that "To anyone interested in the fascinating topic of multiple discoveries in science, and the associated question of why some figures are systematically neglected, Launhardt's case affords a rich example." (1986, p. 123).

In this paper, we will deal with significant and neglected aspects of Laundardt's theoretical contributions to spatial demand and spatial monopoly. It will be shown that the theory of spatial demand and spatial monopolist's economic decision was highly advanced in Chapter 27 of Launhardt's 1885 book. By his mathematical presentation of the spatial monopoly, Launhardt anticipated the spatial monopoly analysis developed and extended by Hoover (1937), Losch (1954), Smithies (1941), Beckmann (1968, 1976), Beckmann and Thisse (1986), Stevens and Rydell (1966), Greenhut, Hwang and Ohta (henceforth GHO)(1975), Greenhut, Norman and Hung (henceforth GNH)(1987), Ohta (1988) and other writers.

2. LAUNHARDT'S SPATIAL MONOPOLY MODEL

Launhardt's contribution to the spatial monopoly model is contained in Chapter 27 of his 1885 book entitled *Market Area for the Sale of Goods*, (1993, pp.141-146). His spatial monopoly model is mainly based on the following assumptions:

A1. A monopolistic firm is located at a point on an unbounded plain where consumers are evenly distributed.

A2. All consumers are identical and each has the same linear demand function.

A3. The freight rate per unit of distance per unit of quantity is constant.[3]

[3] Launhardt noted that "[t]he transport charges are estimated in proportion to the distance the good will travel while – except for the 'differential tariffs' which are of no concern in this context – there are still regular additional cost irrespective of costs growing with the distance, which are independent of these, like packing and loading, unpacking, storage

A4. The firm charges the same mill price to its customers regardless of their location.

A5. Consumers bear transportation costs.

With these assumptions, Launhardt (1993, chapter 27) stated clearly that in the spatial economy

> [t]he price at which goods leave the location of their origin usually experiences an additional charge for the cost of dispatching them from the location of the origin to the location of use or consumption. Due to the increase in price relative to an increasing distance to be covered by transport, demand will decrease, and after a certain distance from the place of origin, deteriorate to zero. (1993, p.141)

Under the assumption of quadratic utility function, he further derived the individual spatial demand as a linear function of the mill price p and distance r from the seller's site. The individual demand function can be specified as:

(1) $q(r) = a - p - tr$

where q is quantity demanded, t is the freight rate, p + tr is the full price paid by the consumer at distance r and a > 0.[4] This individual spatial demand function has been used widely in spatial economics, see Beckmann (1968, 1976), Beackmann and Thisse (1986) GHO (1975), GNH (1987), and Ohta (1988).

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and transport charges. Let us suppose that these charges which are independent of the distance have been included in the price p payable at the location of origin, " (1993, p. 141-142). Note that the price p payable at the location of origin is the mill price.

4. This demand function is consistent with Launhardt's. Launhardt assumed that the utility function for the consumer is $U = \alpha x - \alpha_1 x^2$, $\alpha > 0$, $\alpha_1 > 0$, $x > 0$ and $\alpha > 2\alpha_1 x$. From the optimal choice condition, $MU_x/(p + fz) = w$, he obtained the demand function for an individual as: $x = (\alpha/2\alpha_1) - (w/2\alpha_1)(p+fz) = (w/2\alpha_1)[(\alpha/w) - p - fz]$, where p is the mill price, f is freight rate, z is the consumer's located distance from the seller site and w is the marginal utility of money (income). For details, see Launhardt (1993, p.142, p. 161 and p.172). To ease our analysis, we assume $w/2\alpha_1 = 1$, and $q = x$, $a = \alpha/w$, $f = t$, $r = z$ in Launhardt's linear demand function, hence at once obtain (1). It should be noted

If the consumers are evenly distributed over the market and per surface unit has one consumer[5], the aggregate spatial demand in a market of radius R is

$$(2)\quad Q = 2\pi\int_0^R (a - p - tr)r\,dr = 2\pi\{[(a - p)R^2/2] - (tR^3/3)\}$$

where $\pi = 3.1416\ldots$. Launhardt noted that "the maximum dispatch distance, R*, at which goods become so expensive that demand ceases is determined..", (1993, p. 141). In other words, R* is given by

(3) $a - p - tR^* = 0$, or $R^* = (a - p)/t$

where R* is the maximum radius of a circular market, $a - p > 0$ is Launhardt's dispatch value.[6] Based on (3), Launhardt described the formation of the market area as

> The goods will be sold everywhere within the maximum feasible distance, with *dispatch borders*, so that the *market area* for a good which can be produced in a given location in unlimited quantity will form a circle, given that the respective economic conditions are equal in all directions, with a radius which equals the furthest dispatch distance. (1993, p. 142).[7]

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that the notation used by Launhardt is rather awkward for the modern reader and potentially misleading. In this paper, we use the conventional notation in economic textbooks to replace Launhardt's.

5 . Launhardt assumed that "the buyers are evenly distributed over the market area so that there are n buyers per surface unit," (1993, p. 143). For simplicity, we assume that n = 1.

6 . Launhardt noted that $a - p > 0$ "is the amount by which p at the place of origin stays below the price a, for which the first unit is still purchase by the consumer who can buy," (1993, p.141)

7 . Launhardt further noted that "the size of the market area is $\pi R^{*2} = (\pi/t^2)(a - p)^2$ and under otherwise equal conditions is inversely proportional to the square of the freight rate." (1933, p. 142). It should be noted that this is the well-known Lardner's law of squares, cf., Lardner (1850, p. 258). Recently, Backhause incorrectly claimed that

Substituting $R^* = (a - p)/t$ into (2) gives the aggregate spatial demand function

(4) $Q(R^*) = (\pi/3t^2)(a - p)^3$

or in inverse form,

(4') $p = a - (3t^2Q/\pi)^{1/3}$.

This aggregate demand curve can be illustrated in Figure 1. It is convex to the origin despite the fact that individual demand curve is linear.[8] From (4), Launhardt pointed out that the aggregate demand increases in proportion to the cube of the dispatch value, a – p, and inversely in proportion to the square of the freight rate, t, cf. Launhardt (1993, p. 143).

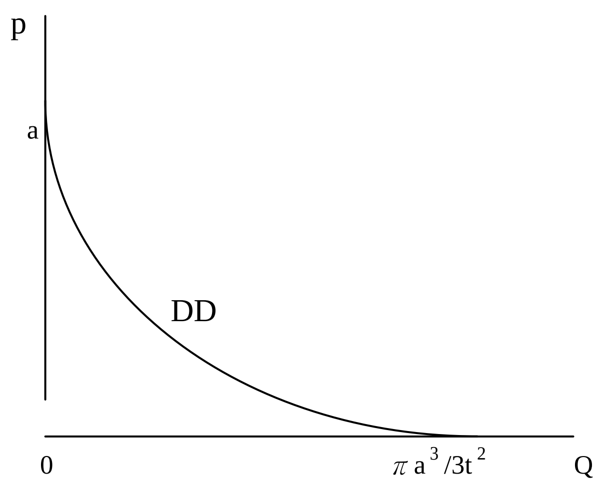

Figure 1. Aggregate Spatial Demand

Launhardt obtained this law under the assumption that the marginal utility is constant. (2000, p. 451). For details, see Shieh (2002).

[8] Taking the first and second order derivative of p with respect to Q gives $dp/dQ = -[3Q(R^*)t^2/\pi]^{-2/3}(t^2/\pi) < 0$ and $d^2p/dQ^2 = 2[3Q(R^*)t^2/\pi]^{-5/3}(t^2/\pi)^2 > 0$. Clearly, the aggregate demand curve is convex to the origin. See also Losch (1954, pp. 106-108), GNH (1987, pp. 9-10) and Ohta (1988, pp. 19-22).

After Launhardt derived the aggregate spatial demand, he set out to investigate the economic decision of a profit-maximizing monopolistic firm. Launhardt didn't present the cost function explicitly, however, from his profit function, we can specify the cost function as:

(5) $C(Q) = cQ$,

where c is the constant marginal cost and Q is monopolist's output. Note that in this case the marginal cost equals average cost, i.e., MC = AC = c.[9]

Since the firm charges the same mill price to all consumers in the market area, the profit function can be specified as:[10]

(6) $\Pi = pQ - C(Q) = p(\pi/3t^2)(a - p)^3 - c(\pi/3t^2)(a - p)^3$

According to Launhardt, the profit-maximizing firm chooses the optimal profit margin when it makes the business decision. Since the mill price is the sum of the average cost and the profit margin, we can specify the profit margin as:

(7) $g = p - c$

where g is the profit margin.

Substituting (7) into (6), the profit function can be rewritten as

(8) $\Pi = (c+g)(\pi/3t^2)(a - c - g)^3 - c(\pi/3t^2)(a - c - g)^3$

$= g(\pi/3t^2)(a - c - g)^3$

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[9] Most writers in spatial economics specify the cost function as: $C(Q) = F + cQ$, where F = fixed cost. However, the specification of cost function as (5) is not uncommon. For instance, see Beckmann and Thisse (1986, p. 31).

[10] Launhardt didn't explain explicitly the difference between the mill price and the marginal cost. He used p to denote the mill price or the marginal cost. To ease our analysis, we use c and p to denote the marginal cost (average cost), and the mill price respectively.

where c, a, t, and π are known, and g is the only choice variable. It should be noted that equation (8) is similar to Launhardt's equation (86) in his 1885 book, cf., (1993, p.144). Laundhardt further ingenuously pointed out that

> If a businessman can exploit the production of certain goods as a *monopoly* then he will determine the profit margin in such a way that his total profit will reach maximum. This most favorable profit margin is arrived at for sales within the market area by differentiation of [the profit function] with respect to g. (1993, p.145)

Following Laundhart, we differentiate (8) with respect to g and set the resulting derivative equal to zero. The first-order condition for profit maximization is

(9) $d\Pi/dg = (\pi/3t^2)(a - c - g)^2(a - c - 4g) = 0$

However, the first-order condition may lead to a minimum rather than a maximum; thus we must check the second-order condition. We can obtain the second derivative by differentiating the first derivative with respect to g

(10) $d^2\Pi/dg^2 = - 4(\pi/3t^2)(a - c - g)^2 < 0$

The second-order condition is always satisfied. Solving (9), we obtain the optimal g as:

(11) $g^* = (a - c)/4$

Substituting (11) into (7) and (8) respectively, we find the optimal mill price and the maximum profit as:

(12) $p^* = c + g^* = (a + 3c)/4$

(13) $\Pi^* = (9\pi/256t^2)(a - c)^4$

Virtually, the results in (11), (12) and (13) are equivalent of Launhardt results (1993, pp. 145-146)[11]. Equation (12) indicates that the profit-maximizing mill price is one-fourth maximal demand price plus three-fourths marginal cost. Beckmann (1968, p. 33, p. 51) and GO (1975, p. 28) obtained the same result. However, they were not aware that Launhardt had already found this result in nineteenth century. [12]

Laundhart didn't consider the optimal output. However, from the above analysis, it is easy to obtain the optimal output. Combining (12) and (4), we obtain the optimal output as

(14) $Q^* = (9\pi/64t^2)\,(a - c)^3$

This result is consistent with GNH (1987, p. 9)

Launhardt's model of spatial monopoly can be shown graphically. Figure 2 illustrates the spatial monopoly model with constant marginal cost. The spatial monopolist faces the aggregate spatial demand curve Q(R*) and the horizontal marginal cost curve MC. The firm will set the profit margin at $g^* = p^* - c$, and the mill price at p* because the firm's maximum profit (Π^*) occurs where MR intersects MC. Figure 2 also shows that the firm will set the optimal output at Q*.

11. Equations (13) and (11) are equivalent of equation (88) and the equation that precedes equation (88) on p. 145 of Launhardt (1993) respectively. Equation (12) is consistent with Launhardt's finding that "if (buyers) were dispersed over a market area, they would have to pay a price of $p + g' = \alpha/4w + 3p/4$", Launhardt (1993, p. 146). Note that $p + g' = \alpha/4w + 3p/4$ is identical to our $p^* = c + g^* = (a + 3c)/4$.

12. The optimal mill price in (12) can also be obtained by specifying the mill price as the choice variable. For details, see Appendix.

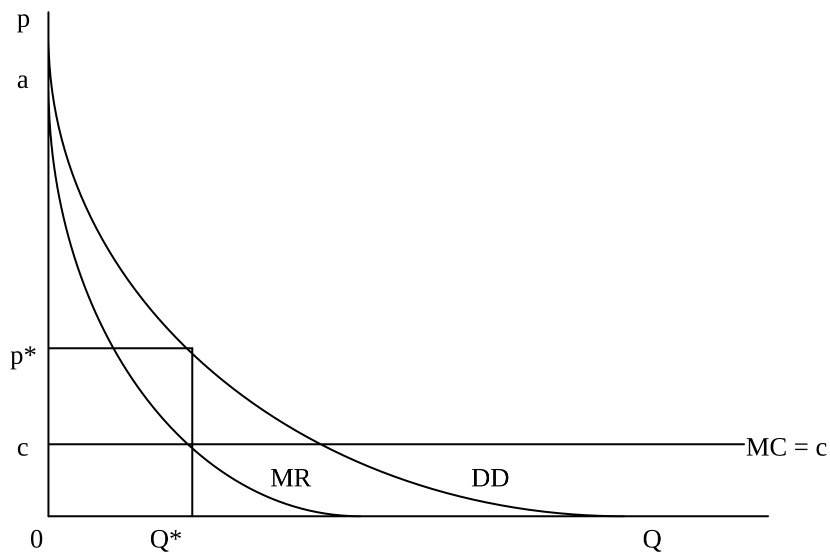

Figure 2. Spatial Monopoly Equilibrium

3. LAUNHARDT'S NON-SPATIAL MONOPOLY CASE

Assume that all consumers are not evenly distributed in the circular market but all are settled at the monopolistic firm's location. Launhardt investigated the economic decision of a non-spatial monopoly. He first calculated the number of consumers, N, in the circular market. Following Launhardt, we substitute the optimal mill price in (12) into (3) and obtain

(15) $R^* = (a - p^*)/t = (3/4t)(a - c)$

where R* is the maximum radius of the circular market. Thus, we can calculate the total number of consumers that are settled with the firm is equal to the area of a circle of radius, R*, i.e.,

(16) $N = \pi R^{*2} = \pi[3(a - c)/4t]^2$

Under the assumption that individual demand functions are identical for all consumers, we obtain the non-spatial aggregate demand simply by multiplying individual demand by the number of consumers:

(17) $Q_o = N(a - p) = (9\pi/16t^2)(a - c)^2(a - p)$

or, in inverse form

(17') $p = a - [(9\pi/16t^2)(a - c)^2]^{-1}Q_o$

Since $[(9\pi/16t^2)(a - c)^2]^{-1}$ is a constant, from (17) and (17'), we can see that in a non-spatial economy the aggregate demand is linear if individual demand is linear, cf. GNH (1987, pp. 6-16), Ohta (1988, pp. 19-24).

With this in mind, the profit function of a non-spatial monopoly can be specified as:

(18) $\Pi_o = pQ_o - cQ_o$

$$= (c + g_o)(9\pi/16t^2)(a - c)^2(a - c - g_o) - c\,(9\pi/16t^2)(a - c)^2(a - c - g_o)$$

$$= g_o(9\pi/16t^2)(a - c)^2(a - c - g_o)$$

where $c + g_o = p$, and g_o is the choice variable. To find the profit-maximizing g_o, we differentiate (18) with respect to g_o and set the derivative equal to zero. The first-order condition is

(19) $d\Pi_o/dg_o = (9\pi/16t^2)(a - c)^2(a - c - 2g_o) = 0$

We further differentiate the first derivative with respect to g_o and obtain the second-order sufficient condition

(20) $d^2\Pi_o/dg_o^2 = - 2(9\pi/16t^2)(a - c)^2 < 0$

Clearly, the second-order condition is always satisfied. Solving (19), we obtain the optimal profit margin as:

(21) $g_o^* = (a - c)/2$

Substituting (21) into (7) and (18) respectively, we find the maximized profit and optimal mill price to be

(22) $p_o^* = (a + c)/2$,

(23) $\Pi_o^* = (9\pi/64t^2)(a - c)^4$

These results are equivalent of Laundhardt's, (1993, pp. 145-146).[13] The optimal mill price is also identical to GNH's (1987, p. 9).

We further substitute (22) into (17) and find the optimal output as

(24) $Q_o^* = (9\pi/32t^2)(a - c)^3$

This result is consistent with GNH (1987, p. 7).

Launhardt's non-spatial monopoly case can also be shown graphically. Figure 3 illustrates the familiar monopoly model with constant marginal cost. The non-spatial monopoly firm faces the linear aggregate demand Q_o and marginal revenue curve MR and the horizontal marginal cost curve MC. The firm will set the profit margin $g_o^* = p_o^*$ - c, the mill price at p_o^* and the optimal output at Q_o^* because the firm's maximum occurs where MR intersects MC.

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[13] Equations (23) and (21) are equivalent of equation (89) and the equation that precedes equation (89) on p. 145 of Launhardt (1993) respectively. Equation (22) is consistent with Launhardt's finding that "In the case where buyers are all in the vicinity of production they will have to pay a price of p + g" = (α/w + p)/2." (1993, p. 146). Note that p + g" = (α/w+ p)/2 is identical to our $p_o^* = c + g_o^* = (a + c)/2$.

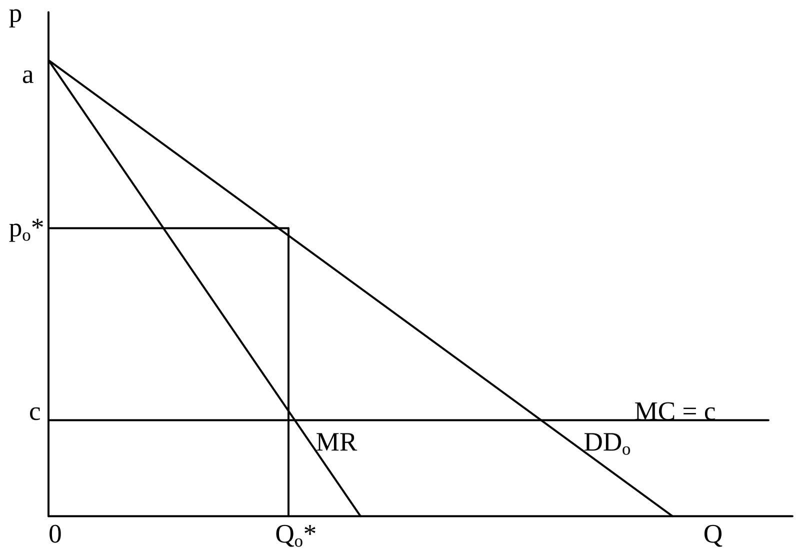

Figure 3. Non-Spatial Monopoly Equilibrium

4. NON-SPATIAL VERSUS SPATIAL MONOPOLY

After analyzing the economic decision of spatial and non-spatial monopoly, Launhardt further investigated the impact of distance and transport cost on mill price and profit. To this aim, following Launhardt, we compare the optimal solution of profit margin, profit and output under spatial and non-spatial monopoly. From (11) and (21), we obtain

(25) $g_o^*/g^* = 2$

It is clear that the profit margin will double if buyers are all settled at the location of the firm. Furthermore, from (12) and (22), we obtain

(26) $p_o^* - p^* = (a+c)/2 – (a + 3c)/4 = (a - c)/4 > 0$, as $a > c$

In other words, the non-spatial monopolistic firm charges a higher mill price than the spatial monopolistic firm when the marginal cost is constant. This result is consistent with GNH (1987, pp. 9-10) and Ohta (1988, p. 25). Ohta also showed that this result applies to the case of rising MC. In the case of the decreasing MC, the spatial monopolistic firm may charge a higher mill price than the non-spatial monopolistic firm, see Ohta (1988, pp. 24-26). Equation (26), in fact, shows that in the spatial economy the transportation cost provides the monopolistic firm with incentive to lower its mill price. A lower mill price is a full blessing to consumers located at the neighborhood of the plant site, cf., Launhardt (1933, p. 146).

Next, let us turn to the profit. Dividing (23) by (13), we obtain

(27) $\Pi_o^*/\Pi^* = (9\pi/64t^2)(a - c)^4/(9\pi/256t^2)(a - c)^4 = 4$

Thus, Launhardt maintained that "[in] a business which is conducted as a monopoly, profit will quadruple provided buyers are not widely dispersed in a market, but are all settled at the location of origin of the goods." (1993, p.146)

Finally, we consider the optimal output. From (14) and (24), we find

(28) $Q_o^*/Q^* = (9\pi/32t^2)(a - c)^3/(9\pi/64t^2)(a - c)^3 = 2$

In other words, the spatial monopoly will produce less output than the non-spatial monopoly. Recently, GHO showed that "the mill price and output level in a spatial monopoly world is lower than in the monopolized market in which cost of distance is zero, ceteris paribus." (1975, p. 680, fn. 17). Obviously, GHO's result had already been anticipated by Launhardt.

5. CONCLUSIONS

The theory of spatial monopoly opens up entirely new ways of viewing the impact of economic space and transportation cost on the economic decision of the firm, for example, spatial price policies, the market area and the plant location problems. Except for Losch's (1954, p. 197) passing note, however, Launhardt's name and work have never mentioned in this connection. In their review of Launhardt's contributions to location theory, Fels (1968), Blaug (1986), Niehans (1987) and Backhaus (2000) mentioned Launhardt's contribution to market area and heterogeneous duopoly, but they didn't mention Launhardt's contribution to the spatial monopoly model. It is generally believed that Hoover (1937), Losch (1954), and Smithies (1941) first published a formal spatial monopoly model, cf., Ponsard (1983, pp. 76-77, p. 91). However, the available evidence as developed above indicates that Launhardt's analysis in Chapter 27 of his 1885 book already contained a sophisticated and original presentation of the spatial monopoly model. Launhardt's contributions must rank as the first attempts at the development of spatial monopoly theory in the nineteenth century literature.

Assume that a monopolistic firm is located at a point on an unbounded plain where consumers are evenly distributed and each consumer has the same linear demand function. Modern economists, like GNH (1987) and Ohta (1988), show that the aggregate demand curve is convex to the origin regardless of the linear individual demand curve. They further show that the profit-maximizing mill price of a spatial monopoly is lower than that of a non-spatial monopoly. Apparently, they were unaware that Launhardt had already discovered the impact of economic space on the aggregate spatial demand curve and the optimal mill price of a spatial monopoly much earlier than Hoover (1937), Losch (1954) and Smithies (1941).

Considering Launhardt's highly original analysis of spatial monopoly that was not surpassed until the 1930's, it is clear that Launhardt's name deserved to be mentioned as the most important early precursor of modern spatial economics.

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APPENDIX

In the text, following Launhardt, we consider the choice variable of the monopolistic firm is the profit margin. However, in spatial economics, most economists consider the mill price as the choice variable under f.o.b. pricing policy. In this appendix, we consider the case in which the choice variable is the mill price.

According to Launhardt’s spatial monopoly model, the profit maximization problem under f.o.b. pricing can be specified as:

$$\text{(A1)} \quad \text{Max } \Pi = (p - c)Q$$

$$= (p - c)2\pi\int_0^R (a - p - tr)r\,dr$$

where p is the choice variable.

The optimal mill price p is determined by the first-order condition for a maximum:

$$\text{(A2)} \quad d\Pi/dp = 2\pi[(a - 2p + c)\int_0^R r\,dr - t\int_0^R r^2 dr] = 0$$

or

$$\text{(A3)} \quad p^* = (1/2)\{(a + c) - t[\int_0^R r^2 dr / \int_0^R r dr]\} = (1/2)[(a + c) - t(2/3)R^*]$$

The second-order condition for a maximum is satisfied as

$$\text{(A4)} \quad d^2\Pi/dp^2 = - 4\pi\int_0^R rdr < 0$$

Substituting $R^* = (a - p^*)/t$ into (A3), we can solve the optimal mill price as:

$$\text{(A5)} \quad p^* = (1/4)(a + 3c)$$

Equation (A5) is identical to equation (12) in the text. It is worth mentioning that R^* is the maximum radius of the market area at which demand falls to zero.